AddMore Federation

Calculation Policy

Here at Adderley CE Primary School we are passionate about Maths! As Maths is a core subject, it is taught daily - both in discrete lessons and, whenever possible, incorporated into other areas of the curriculum. The purpose of Mathematics at Adderley is to ensure our children develop an ability to solve problems, to reason, to think logically and to work systematically and accurately.

We believe that all our children are challenged and encouraged not only to meet the expectations of the National Curriculum, but to also excel in Maths and enjoy the journey that it takes them on! Through our approach to the teaching of Mathematics we encourage resilience and reflection by promoting a 'Growth Mindset' approach and the confidence to believe that everyone can achieve in Maths. We have adopted and adapted the White Rose Mixed Age schemes of work, to allow us to teach maths at the appropriate levels across our mixed aged classes and our unique setting. New mathematical concepts are introduced using a 'Concrete, Pictorial and Abstract' (CPA) approach, this enables all children to experience practical hands-on learning when discovering new mathematical topics, allowing children of all learning styles to access and enjoy maths. This method also allows our children to have clear models and images to aid their understanding, which they will then take with them throughout their mathematical journeys.

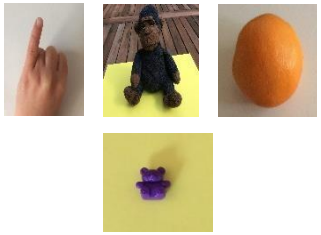
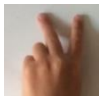
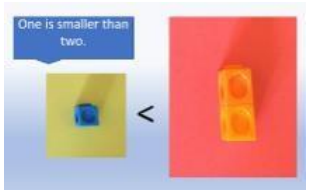
We are and ambitious and academic school that strives, through its leaderships and curriculum, to ensure we equip our children with the knowledge and skills they require to become confident and capable mathematicians.

"The essence of mathematics is not to make simple things complicated, but to make complicated things simple."

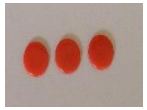
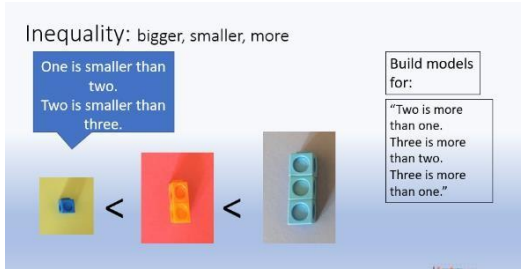
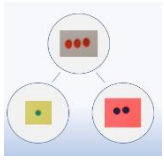
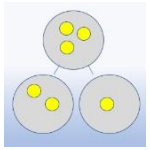
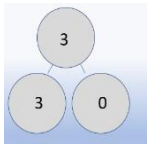
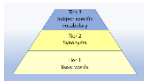
Stan Gudder

In addition to our CPA Teaching sequence we identify the key skills and words and phrases that children need to understand and use if they are to make good progress in mathematics. Prior learning is built upon and there are regular opportunities to revisit prior learning to ensure durability of knowledge and skills. We support this by using a three-tiered approach which identifies basic vocabulary as well as subject specific language and allows opportunity for developing fluency as well as deep thinking; encouraging all of our mathematicians to explain, qualify and deepen their thinking and understanding of key mathematical concepts.

Addition and Subtraction

Nursery: 22-36 months Selects a small number of objects from a group when asked, for example, 'please give me one', 'please give me two'. Creates and experiments with symbols and marks representing ideas of number Begins to make comparisons between quantities. Uses some language of quantities, such as 'more' and 'a lot' Knows that a group of things changes in quantity when something is added or taken away.				
Representations	Key knowledge and vocabulary	Concrete & pictorial Conceptual modelling	Abstract Skills and knowledge	Application across the environment
    	Concepts of quantity, equality and inequality. Modelling combining sets of small quantities. Modelling adding to a quantity to make it bigger. Removing objects from a set to show the amount is now smaller.	Natural materials and physical objects in all environments. Pictures to show one or two items. Objects and resources to physically represent a quantity. Images and pictures to represent a small quantity. Using dishes/hoops to make quantities of different values that visually show one set has more than the other. Images of quantities to compare. Which has more?	Spoken number names. *One, once, alone, first.* Mark making and graphics to represent a small number in the context of play. Mark making and graphics to represent a small quantity to compare in the context of play.	Wonderful one and terrific two displays. Hiding objects find one of, or lots of in the sand, across the setting. Matching one item to another then to one image. Repeat with two. Snack time: one piece of fruit to one person, two pieces each Problem solving: "We need one/two each how can we sort the bears?"

Nursery/Reception: 30 - 50 months
Knows that numbers identify how many objects are in a set.
Beginning to represent numbers using fingers, marks on paper or pictures.
Sometimes matches numeral and quantity correctly.
Compares two groups of objects, saying when they have the same number.
Separates a group of three or four objects in different ways, beginning to recognise that the total is still the same.
Shows an interest in representing numbers.

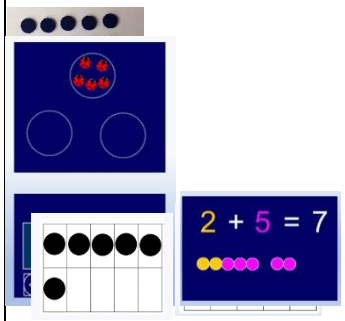
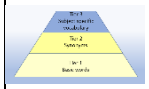
Representations	Key Vocabulary	Key knowledge	Concrete & pictorial Conceptual modelling	Abstract Skills and knowledge	Application across the environment
     	Layers of vocabulary  **Appendix** Beck's Tiers of Vocabulary **Basic to subject specific (Beck's Tiers):** Add, more, and, make, sum, total, altogether, double, how many **Instructional vocabulary:** Listen, join in, say, start from, look at, carry on	Concepts of cardinality, equality, inequality and rearranging the same quantity. Counting to 3. One to one correspondence. Knowing how many are in the set. Comparing numbers 1,2 and 3 – 'bigger' and 'smaller' Stable ordering numbers 1 to 3. 3 is made up of 2 and 1. Using counting strategies and subitising to identify the number of concrete objects in the set.	Natural materials and physical objects in all environments to count. (cardinality) Pictures to show a quantity that can be counted. Use fingers to show small amounts. Images and pictures to represent a small quantity. Resources that match a numeral to a quantity. E.g. a number track, digits cards with numerals and quantities represented.	Represent a quantity by drawing. Mark making and graphics to represent a small quantity and attempts at numerals. Mark making and drawings to replicate the concrete and pictorial model. With models, attempts to write numerals and continue to mark make.	Construction. What can you make with 3 / 4 bricks? Small world. Put three carriages on the train. How many cars are in the car park? How many skittles have you knocked over? Mark making and graphics to represent a small number in the context of play.

Reception: 40 - 60 months

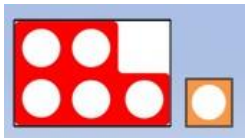
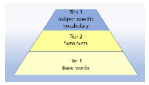
Counts up to three or four objects by saying one number name for each item. Counts objects to 10 and beginning to count beyond 10.

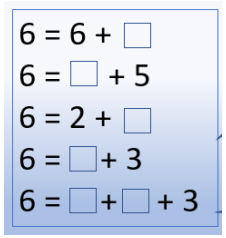
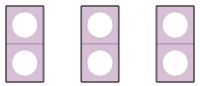
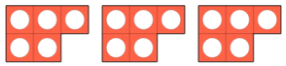
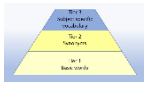
Selects the correct numeral to represent 1 to 5, then 1 to 10 objects. Uses the language of 'more' and 'fewer' to compare two sets of objects. Finds the total number of items in two groups by counting all of them. Says the number that is one more than a given number.

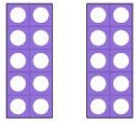
In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting.

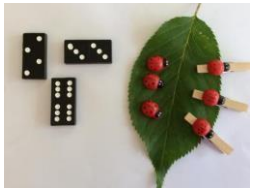
Representations	Key Vocabulary	Key knowledge and vocabulary	Concrete & pictorial Conceptual modelling	Abstract Skills and knowledge	Application across the environment
	Layers of vocabulary  Appendix Beck's Tiers of Vocabulary Basic to subject specific (Beck's Tiers): Add, more, and, make, sum, total, altogether, double, how many more to make, how	Number structure. Equality, inequality. Partitioning and recombing. Subitising to 5. 5 as an anchor. Modelling the combining of sets, recognising that the quantity has increased. Using counting strategies and subitising to identify the number of concrete/pictorial objects in the set	Natural materials, physical objects and mathematical resources e.g. counters in all environments to count accurately. (cardinality). To 10 and beyond. Pictures to show a quantity that can be counted then to 10 and beyond. Resources that match a numeral to a quantity Models of mathematical counting resources to show the more or fewer. Using a	Represent a quantity by drawing or by using graphics. (using drawings to show a resource) Mark making and graphics to represent numbers to 10 and beyond in their play. Graphics and attempts at numerals in the correct orientation.	Malleable play: problem solving 'Let's put 5 cherries on the cakes.' 'How will you put your 5 candles on the two cakes?' Role play: problem solving Each shelf in the shop must have 5 or more items to sell. How shall we arrange the items? Find items in the sand. 3 shells and 2 fish. How many items altogether?

	many are left, how many have gone? Instructional vocabulary: Listen, join in, say, start from, look at, carry on, what comes next, find, choose, talk about		number track or line to show one more than a given number	Mark making and numerals to replicate the concrete and pictorial model. Graphics and numerals to show the addition	

Reception: ELG 2018 Numbers to 20: place them in order and say which number is one more or one less than a given number Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer They solve problems, including doubling, halving and sharing.					
Representations	Key Vocabulary	Key knowledge and vocabulary	Concrete & pictorial Conceptual modelling	Abstract Skills and knowledge	Application across the environment
3 + ☐ = 6 1 + 5 = ☐ ☐ + 0 = 6 3 + 3 = ☐ 5 + ☐ = 6 	Layers of vocabulary  **Appendix** Beck's Tiers of Vocabulary **Basic to subject specific (Beck's Tiers):** Add, more, and, make, sum, total, altogether, double, how many more to	Number structure. Equality, inequality. Partitioning and recombing. Subitising to 5. 5 as an anchor. Modelling the combining of sets, recognising that the quantity has increased. Using counting strategies and subitising to identify the number of	Natural materials, physical objects and mathematical resources e.g. counters in all environments to count accurately. (cardinality). To 10 and beyond. Pictures to show a quantity that can be counted then to 10 and beyond. Resources that match a numeral to a quantity	Represent a quantity by drawing or by using graphics. (using drawings to show a resource) Mark making and graphics to represent numbers to 10 and beyond in their play. Graphics and attempts at numerals in the correct orientation. Mark making and numerals to replicate the	Malleable play: problem solving 'Let's put 5 cherries on the cakes.' 'How will you put your 5 candles on the two cakes?' Role play: problem solving Each shelf in the shop must have 5 or more items to sell. How shall we arrange the items? Find items in the sand. 3 shells and 2 fish. How many items altogether?

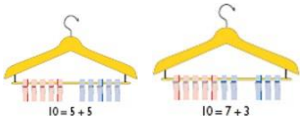
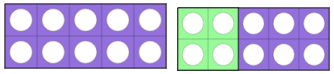
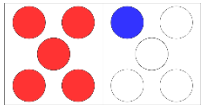
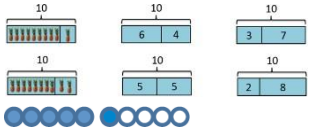
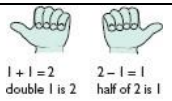
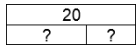
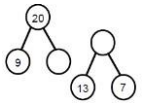
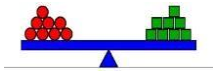
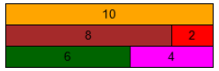
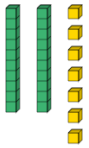
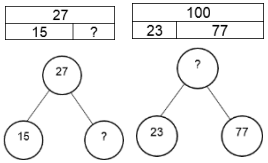
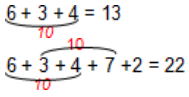
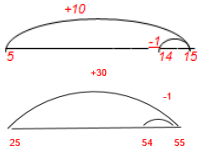
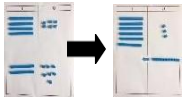
	make, how many are left, how many have gone? One less, two less, ten less, the difference between, odd and even. **Instructional vocabulary:** Listen, join in, say, start from, look at, carry on, what comes next, find, chose, talk about, repeat, tell me, describe, complete	concrete/pictorial objects in the set .	Models of mathematical counting resources to show the more or fewer. Using a number track or line to show one more than a given number	concrete and pictorial model. Graphics and numerals to show the addition	
	Reception: ELG 2018 Numbers to 20: place them in order and say which number is one more or one less than a given number Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer **They solve problems, including doubling, halving and sharing.**				
Representations	Key Vocabulary	Key knowledge	Concrete & pictorial Conceptual modelling	Abstract Skills and knowledge	Application across the environment
 Counting in 2s 	Layers of vocabulary  **Appendix** Beck's Tiers of Vocabulary	Knowing that groups of the same quantity are added together. That is what makes a double. The quantity divided into two equal groups. Halving.	Natural materials, physical objects and mathematical resources e.g. counters in all environments to double, share, group and half accurately.	Represent a quantity by drawing or by using graphics. (using drawings to show a resource) Graphics and numerals to show the double/halving/grouping and sharing used.	In small world play: All the animals in the enclosures are doubles. How many lions will there be etc? Doubles shop Everything in the shop has to be double.

Counting in 5s  Double 10 is 20.  8 divided in to groups of 2.  4 shared equally into two groups.	**Basic to subject specific (Beck's Tiers):** Add, more, and, make, sum, total, altogether, double, how many more to make, how many are left, how many have gone? One less, two less, ten less, the difference between, odd and even. Equals, share, groups of, halve and half **Instructional vocabulary:** Listen, join in, say, start from, look at, carry on, what comes next, find, choose, talk about, repeat, tell me, describe, complete, pattern, remember, ring, work out, check, another way	Sharing and grouping. Sharing is where you take a quantity and count out into how many equal groups you want. Grouping is where you take the quantity and make the groups (of two, or three etc)	Modelling and demonstrating groups of and shared quantities. Showing that the quantity has increased when doubled and reduced when halved.		Snack time How will we share the fruit so that we can have half each?

 To halve the apple it would be cut into two equal pieces To halve the satsuma we would count the segments and share them equally.  Double the number of ladybirds. This show half the number of lady birds sitting on the leaf.  Doubling and halving.					

Addition- KS1

EYFS	**Reception: ELG 2020** • Have an understanding of number to 10, linking names of numbers, numerals, their value, and their position in the counting order. • Subitise (recognise quantities without counting) up to 5. • Automatically recall number bonds for numbers 0-5 and *for 10*, including corresponding partitioning facts. • Automatically recall double facts up 5+5 • Compare sets of objects up to 10 in different contexts, considering size and difference • Explore patterns of numbers within numbers up to 10, including evens and odds.	
Year	1	2
Layers of vocabulary  **Appendix** Beck's Tiers of Vocabulary	**Basic to subject specific (Beck's Tiers):** +, add, more plus make, sum, total altogether score double, near double one more, two more… ten more how many more to make…? how many more is… than…? how much more is…? **Instructional vocabulary:** start from, start with, start at look at point, to show me	**Basic to subject specific (Beck's Tiers):** +, add, addition, more, plus make, sum, total altogether score double, near double one more, two more... ten more... one hundred more how many more to make…? how many more is… than…? how much more is…? **Instructional vocabulary:** tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of… show how you…
NC 2014	Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.	Using concrete objects and pictorial representations, including those involving numbers, quantities and measures applying their increasing knowledge of mental and written methods
	Concrete, pictorial, abstract	Concrete, pictorial, abstract

Developing Conceptual/ Procedural Understanding	**Number bonds**  We have 10 pegs on the coathangers, how can we split them into 2 groups? Is there another way? How can we be sure we have got them all?  Ten Frames Hungarian frames  Use the pattern to complete the number sentences.  Use bonds of 10 to calculate bonds of 20.	  **Recognise small quantities**  **Count on**  **Count on, on number track in 1s.** **Develop knowledge of fact families.**  10 = 3+7 10 = 7+3 10 – 7 = 3 10 – 3 = 7  20 = 3 + 17 20 = 17 + 3 20 – 3 = 17 20 – 17 = 3	**Whole-part model**   Fill in the missing numbers **Balance image for concept of equality.**  9 = 9 9 = 8 + 1 9 = 7 + 2 8 + 1 = 7 + 2  10 = 10 10 = 8 + 2 10 = 6 + 4 8 + 2 = 6 + 4	**Base 10**  **Whole-part model**  Fill in the missing numbers All answers to be recorded in a number sentence following any informal recording. **Adding more than two numbers** Strategy to include looking for facts or bonds that are useful e.g. bonds up to and including 10, doubles or adding 10 to a given number.  Record thinking.	**Adjustment strategy** 5 + 9 = 5 + 10 – 1= 14  (Round and adjust) **Doubles then near doubles** 5 + 6 = 5 + 5 + 1 = 11 7 + 8 = 8 + 8 - 1 = 15 47+50 = **Re-arranging** 18+4 = Tell me what you know about 4, e.g. 3+1, 2+2 18+4= Rearrange the 4 into 2+2 18+2+2= 20+2 =22 59+24 = Partition the 24 into 20 +4 and rearrange the 4 into 1+3. So 59+24= 59+20+1+3 = 59+1+20+3 = 83	**Partition and recombine** Record partitioned steps in number sentences then add mentally. 40+20=60 6+7 =13 60+13=73 Moving on to: 46 + 27 = 60 + 13 = 73  Regrouping the 10. **Balance in the equation** 14 = 8 + 6, 7+6=8+5 □= 13+9 3+ □+6 =16 14+ ◇= 15+27 **Decision making** Using statements such as: Ben did 14 + 9 = 23 How could he have done it?

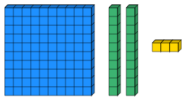
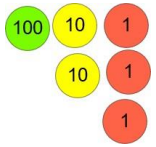
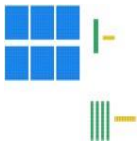
Addition- KS2

<table>
<tr><td>KS1</td><td colspan="5">Pupils should practise addition to 20 and within to become increasingly fluent. They should use the facts they know to derive others, e.g using 7 + 3 = 10 to find 17 + 3= 20, 70 + 30 = 100
They should use concrete objects and practical apparatus, such as bead strings and number lines to explore additions including missing numbers.
Use pictorial representations such as bar models and whole part diagrams to show additive relationships.
100 squares could be used to explore patterns in calculations such as 74 +11, 77 + 9 encouraging children to think about ‘What do you notice?’ where partitioning or adjusting is used.

Pupils should learn to check their calculations, by using the inverse.
They should continue to see addition as both combining groups and counting on.
They should use Dienes to model partitioning into tens and ones* and learn to rearrange numbers in different ways e.g. 23 = 20 + 3 = 10 + 13.
Show understanding that adding zero leaves a number unchanged.</td></tr>
<tr><td>Year</td><td colspan="3">3</td><td colspan="2">4</td></tr>
<tr><td>Layers of vocabulary
 
Appendix
Beck’s Tiers of Vocabulary</td><td colspan="3">Basic to subject specific (Beck’s Tiers):
+, add, addition, more, plus make, sum, total altogether score double, near double one more, two more... ten more... one hundred more how many more to make…? how many more is… than…? how much more is…?

Instructional vocabulary:
explain your method explain how you got your answer give an example of… show how you… show your working</td><td colspan="2">Basic to subject specific (Beck’s Tiers):
add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make…?

Instructional vocabulary:
calculate, work out, solve investigate, question answer check</td></tr>
<tr><td>NC 2014</td><td colspan="3">Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction.</td><td colspan="2">Add and subtract numbers with up to 4 digits using the formal written method of columnar addition and subtraction where appropriate. Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.</td></tr>
<tr><td>Developing Conceptual/ Procedural Understanding</td><td>Near doubles
13+14 =
Double 13= 26
26+1 =27</td><td>Start with least significant digit</td><td>Columnar addition</td><td>Using known facts
40 + 80 = 120 using 4 + 8 = 12</td><td>Columnar addition</td><td>Columnar addition (decimals) in contexts such as money and measurement</td></tr>
</table>

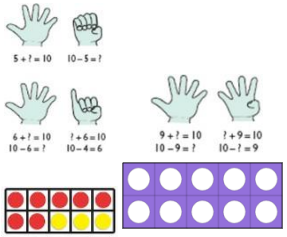
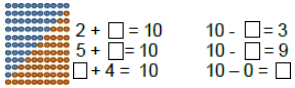
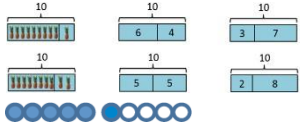
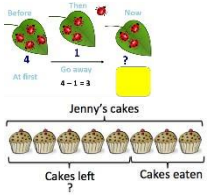
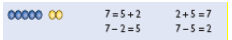
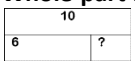
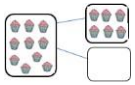
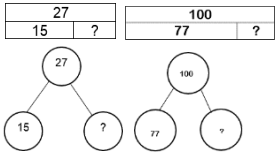
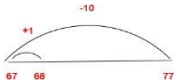
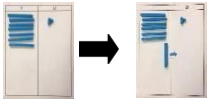
	or Double 14 =28 28-1=27 **Using known facts** 40 + 80 = 120 using 4 + 8 = 12 So 400 + 800 = 1200 **Remodelling strategy** 243 + 198 241 + 200 = 441 **Place value materials to represent 3 digit numbers** Base 10 and then place value counters.  	67 + 24 11 (7+4) + 80 (60+20) 91 "7 add 4 equals 11 and 60 add 20 equals 80. 1+ 0 = 1 and 1 ten + 8 tens = 9 tens"  625 + 48 13 (5+8) 60 (20 + 40) +600 (600 + 0) 673 All language in the context of the place value and the mental addition of the totals to be done in any order.	625 + 48 673 1 **Teach the carried digit.** **Representing problems** There are 334 children at Springfield School and 75 at Oak Nursery. How many children are there altogether?	So 400 + 800 = 1200 and 4000+8000=12,000 **Remodelling strategy** 3548 + 1998 3546 + 2000 = 5546 **Place value materials to represent calculations**	587 + 475 1062 1 1 "7 add 5 equals 12. That's 2 units and 1 ten to carry over. 80 add 70 equals 150 and the1 ten to carry makes 160. That's 6 tens and 100 to carry over. 500 add 400 equals 900 and the 1 hundred to carry makes 1000" 7648 +1486 14 (8+6) 120 (40+80) 1000 (600+400) + 8000 (7000+1000) 9134 7648 + 1486 9134 1 1 1	12.45 7.36 + 24.50 44.31 1 1 1 **Representing problems** There are 259 more boys than girls in Lucy's school. If there are 789 girls, how many pupils are there altogether? ? 

Year	5		6	
Layers of vocabulary  **Appendix** Beck's Tiers of Vocabulary	**Basic to subject specific (Beck's Tiers):** add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make…? **Instructional vocabulary:** **put, place arrange, rearrange change, change over split, separate**		**Basic to subject specific (Beck's Tiers):** add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make…? **Instructional vocabulary:** put, place arrange, rearrange change, change over adjusting, adjust split, separate carry on, continue, repeat what comes next? predict describe the pattern, describe the rule find, find all, find different investigate	
NC 2014	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction). Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.		Solve problems involving addition, subtraction, multiplication and division.	
Developing Conceptual/ Procedural Understanding	**Columnar addition** Include calculations involving more than 2 numbers and carrying figures >1. 25567 16397	**Representing problems** If 2541 is the answer, what's the question? - Can you create three addition calculations? - Can you create three subtraction calculations? - Did you use a strategy?	**Columnar addition** Include calculations with up to 3 'empty columns'. 128.7 + 3.014 128.700 +3.014 131.714	**Representing problems** 7208 females attended a concert as well as 8963 males. There were originally 20000 seats on sale. How many empty seats were there at the concert?

	+15984 57948 1 1 2 1 Include calculations with 'empty columns'. 124.9 + 7.25 124.90 + 7.25 132.25 1 1		1	

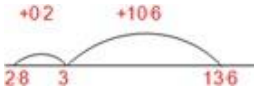
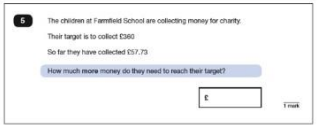
Subtraction- KS1

EYFS	**Reception: ELG 2020** • Have an understanding of number to 10, linking names of numbers, numerals, their value, and their position in the counting order. • Subitise (recognise quantities without counting) up to 5. • Automatically recall number bonds for numbers 0-5 and *for 10*, including corresponding partitioning facts. • Automatically recall double facts up 5+5 • Compare sets of objects up to 10 in different contexts, considering size and difference • Explore patterns of numbers within numbers up to 10, including evens and odds.	
Year	1	2
Layers of vocabulary  **Appendix** Beck's Tiers of Vocabulary	**Basic to subject specific (Beck's Tiers):** take away, distance between, difference between, less than. How many more? How much greater? How many fewer? how much more is…? – subtract, take (away), minus, leave, how many are left/left over? how many have gone? one less, two less, ten less… how many fewer is… than…? how much less is…? difference between half, halve = equals, sign, is the same as **Instructional vocabulary:** start from, start with, start at look at point, to show me	**Basic to subject specific (Beck's Tiers):** subtract, subtraction, take (away), minus leave, how many are left/left over? one less, two less… ten less… one hundred less how many fewer is… than…? how much less is…? difference between half, halve = equals, sign, is the same as tens boundary difference, partition, rearrange, inverse, place value **Instructional vocabulary:** tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of… show how you…
NC 2014	Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.	Using concrete objects and pictorial representations, including those involving numbers, quantities and measures applying their increasing knowledge of mental and written methods
	Concrete, pictorial, abstract	Concrete, pictorial, abstract

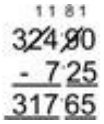
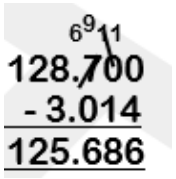
Developing Conceptual/ Procedural Understanding	**Number bonds**  Ten Frames Difference between 7 and 10.  $2 + \square = 10$ $10 - \square = 3$ $5 + \square = 10$ $10 - \square = 9$ $\square + 4 = 10$ $10 - 0 = \square$ Use the pattern to complete the number sentences.  6 less than 10 is 4. **Count out, then count how many are left. Remove from the set.** 7 – 4 = 3 	**Count back on a number track.** 15 – 6 = 9  **Difference between.** 13 - 8 = 8+ __= 13 **Subtraction-take away**  8-3=? **Subtraction-finding the difference**  How many more cakes does Peter have than Jenny? 8-3=?	**Develop knowledge of fact families.**  **Whole-part model**     Fill in the missing numbers	**Whole-part model**  Fill in the missing numbers All answers to be recorded in a number sentence following any informal recording. **Adjustment strategy** 77 - 9 = 77-10 +1 =67+ 1 =68  **(Round and adjust)** What is the nearest 10? 55 – 27 = 55 – 30 +3 =25 + 3 = 28 91 – 48 = 91-50 +2=41 +2 =43	**Re-arranging** 35 - 8 = Tell me what you know about 8, e.g. 2 + 6, 5 + 3 35 - 8= Rearrange the 8 into 5 + 3 So 35 - 5 - 3= 30 - 3 =27 55 - 27 = Partition the 27 into 20 +7 and rearrange the 7 into 5 + 2. So 55 – 27 = 55 -20 -5 -2 = 35 – 5 - 2 = 28 **Taking away and exchanging** 73 – 46 =  What do we know Exchange to make about 76? '60 and 13'.  73 - 46 = 27 Now take away the 46.	**Subtract mentally pairs of multiples of 10 using known facts** 60 – 20 = 40 because 6 – 2 = 4 **Partitioning of the second number strategy** 74 – 47 74 – 40 = 34 34 – 4 – 3 = 27 **Balance in the equation** 35 - = 31 - 12 =34 20 - = 14 - 3 (Open-ended) 18 - = 15 - **Decision making** 27 - = 12 Sam works out 27 – 15 = 12. How could he have done this?

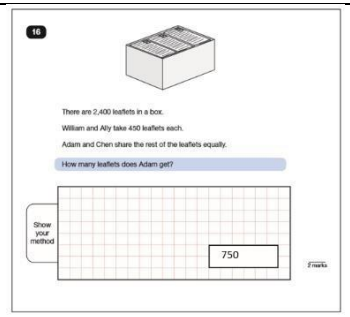
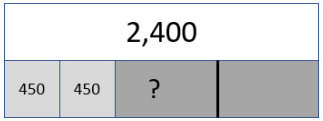
Subtraction- KS2

KS1	Pupils should practise subtraction to 20 and within to become increasingly fluent. They should use the facts they know to derive others, e.g using 10 - 7 = 3 and 7 = 10 - 3 to calculate 100 - 70 = 30 and 70 = 100 - 30. Know the effect of zero. As well as number lines, 100 squares could be used to model calculations such as 74 – 11, 77 – 9 or 36 – 14, where partitioning or adjusting are used. Pupils should learn to check their calculations, including by adding to check. They should continue to see subtraction as both take away and finding the difference and should find a small difference by counting up. They should use Dienes to model partitioning into tens and ones* and learn to partition numbers in different ways e.g. 23 = 20 + 3 = 10 + 13.					
Year	3			4		
Layers of vocabulary  **Appendix** Beck's Tiers of Vocabulary	**Basic to subject specific (Beck's Tiers):** subtract, subtraction, take (away), minus leave, how many are left/left over? one less, two less… ten less… one hundred less how many fewer is… than…? how much less is…? difference between half, halve = equals, sign, is the same as tens boundary, hundreds boundary exchange, carried digits **Instructional vocabulary:** explain your method explain how you got your answer give an example of… show how you… show your working			**Basic to subject specific (Beck's Tiers):** subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? difference between half, halve how many more/fewer is… than…? how much more/less is…? equals, sign, is the same as tens boundary, hundreds boundary, inverse exchange, carried digits **Instructional vocabulary:** calculate, work out, solve investigate, question answer check		
NC 2014	Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction. Least significant digit is always dealt with first to establish if the exchange is needed.			Add and subtract numbers with up to 4 digits using the formal written method of columnar addition and subtraction where appropriate. Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.		
Developing Conceptual/ Procedural Understanding	**Subtract mentally pairs of multiples of 100 using known facts** 600 – 200 = 400 because 6 – 2 = 4	**Start with least significant digit – decomposition**	**Columnar subtraction** 6 14 1 ~~7~~ ~~5~~ 4 - 2 8 6 4 6 8 **Emphasis on language of place**	**Subtract mentally pairs of multiples of 1000 using known facts** 6000 – 2000= 4000 because 6 – 2 = 4	**Columnar subtraction** 2344 -187 2 1 3 1 ~~2344~~ -187 2157	**Representing problems** Check the answer to the following calculations using the inverse. Show all your working.

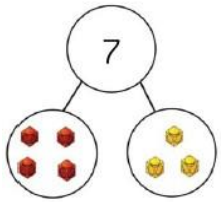
	Remodelling strategy (keeping the difference the same) 502 – 198 **504 – 200 = 304** **Re-arranging** **Use of apparatus to understand rearrangements, e.g. 55 as 40 and 15(not as part of calculations).** **Place value materials to represent numbers in calculations**  	**81 = 80 1** **- 57 50 7** **__ _____** **81 = 70 11** **- 57 50 7** **24_ 20 4_** **"1 subtract 7 is tricky so I will rearrange 81 into 70 and 11. 11 subtract 7 equals 4 and 70 subtract 50 equals 20. 20 and 4 make 24."** **754 700 50 4** **- 86 80 6** **___** **_____** **754 600 140 14** **- 86 80 6** **668 600 60 8** **"It's tricky to take 6 from 4 and 80 from 50. I need to rearrange the number. I will exchange one ten from 50 which leaves**	**value, i.e. 14 units subtract 6 units, 14 tens subtract 8 tens, and 6 hundreds subtract 2 hundreds.** **Representing problems There are 386 pupils at Oak Primary. If 79 pupils have sandwiches, how many have dinners?** **386** **? 79**	**Remodelling strategy (keeping the difference the same)** **3548 - 1998** **3550 – 2000 = 1550** **Find the difference strategy** **13 .6 – 2.8 =**  **13.6 – 2.8 = 10.8** **Place value materials to represent calculations.**	**6467 – 2684** 5 13 1 ~~6~~~~4~~67 - 2684 3783 **Columnar subtraction (decimals) in contexts such as money and measurement** **32.34 – 14.18** 2 1 2 1 ~~3~~2,34 -14.18 18.16	  **2456- 734 = 1822**

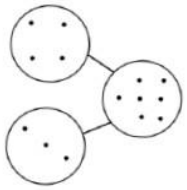
		40 and makes 14 in the units. 40 to subtract 80 is tricky. I will exchange one hundred from 700 and make 140. 14 subtract 6 equals 8. 140 subtract 80 equals 60 and 600 subtract 0 equals 600.”				

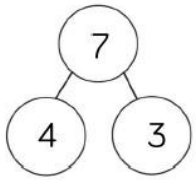
Year	5		6	
Layers of vocabulary  **Appendix** Beck's Tiers of Vocabulary	**Basic to subject specific (Beck's Tiers):** subtract, subtraction, take (away), minus, leave, how many are left/left over? ten less… one hundred less how many fewer is… than…? how much less is…? difference between half, halve = equals, sign, is the same as tens boundary, hundreds boundary, inverse, units boundary, tenths boundary exchange, carried digits **Instructional vocabulary:** put, place arrange, rearrange change, change over adjusting, adjust split, separate		**Basic to subject specific (Beck's Tiers):** subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? difference between half, halve how many more/fewer is… than…? how much more/less is…? equals, sign, is the same as tens boundary, hundreds boundary, units boundary, tenths boundary, inverse **Instructional vocabulary:** put, place arrange, rearrange change, change over adjusting, adjust split, separate carry on, continue, repeat what comes next? predict describe the pattern, describe the rule find, find all, find different investigate	
NC 2014	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction). Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.		Solve problems involving addition, subtraction, multiplication and division.	
Developing Conceptual/ Procedural Understanding	**Columnar subtraction**  Include calculations with 'empty columns'. 324.9 - 7.25 	**Representing problems** Kangchenjunga is the third highest mountain in the world at 28,169 feet above sea level. Lhotse is the fourth highest at 27,960 feet above sea level. Find the difference in heights mentally. Keeping the difference, the same to make the numbers easier to calculate with.	**Columnar subtraction** Include calculations with up to 3 'empty columns'. 128.7 - 3.014 	**Representing problems** Katie was given the calculation below 47326 – 1900 = She said "I will just take off 2000 then subtract another 100 so my answer is 45126." Is she correct? Would you use her method? Explain your answer

		122, 456 – 11,999 **122, 457 – 12,000**		 

Part-Whole Model

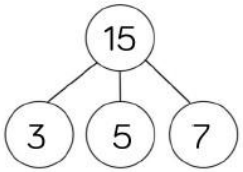


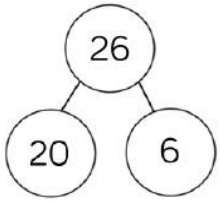


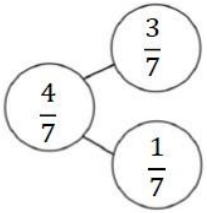


$7 = 4 + 3$ $\quad$ $7 - 3 = 4$

$7 = 3 + 4$ $\quad$ $7 - 4 = 3$







Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

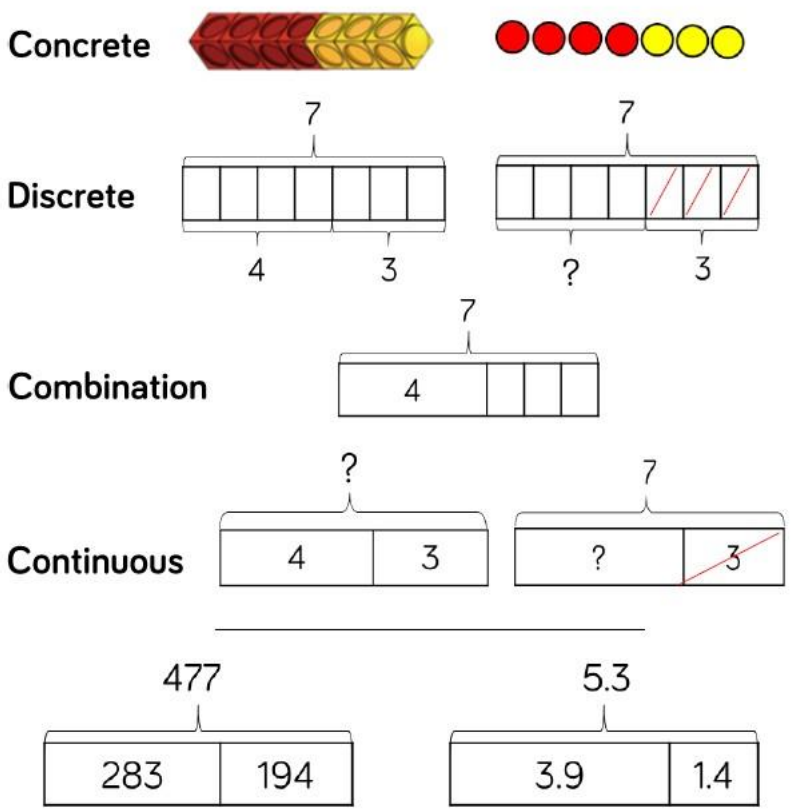
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

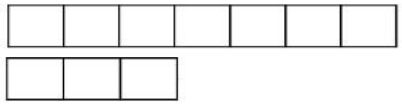
The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

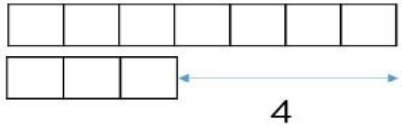
In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (multiple)

Discrete

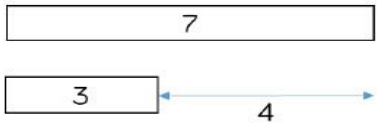


$7 + 3 = 10$

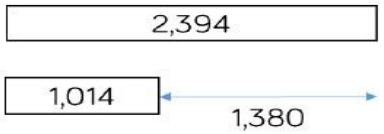


$7 - 3 = 4$

Continuous



$7 - 3 = 4$



$2{,}394 - 1{,}014 = 1{,}380$

Benefits

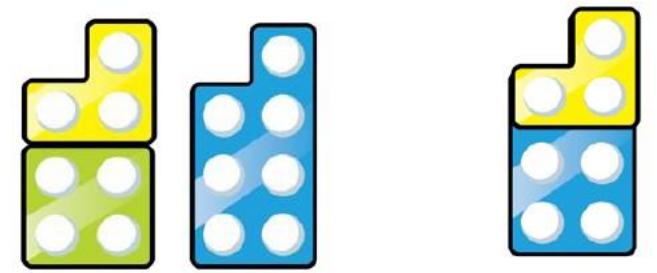
The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

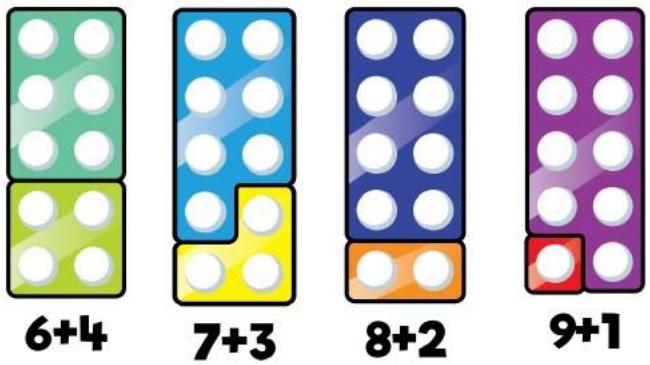
Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Number Shapes





Benefits

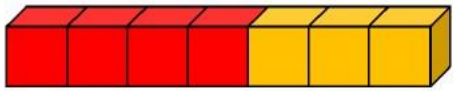
Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

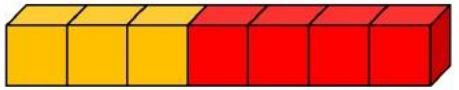
When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

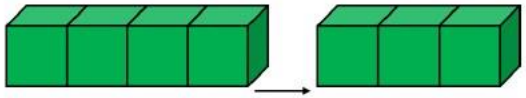
Cubes



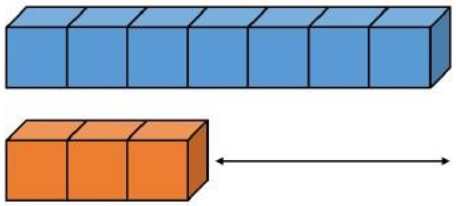
$7 = 4 + 3$



$7 = 3 + 4$



$7 - 3 = 4$



$7 - 3 = 4$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

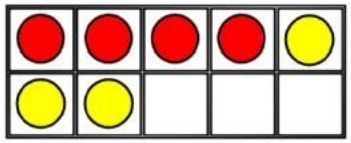
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

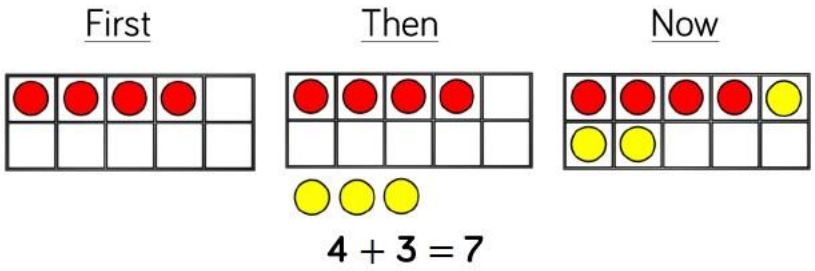
Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

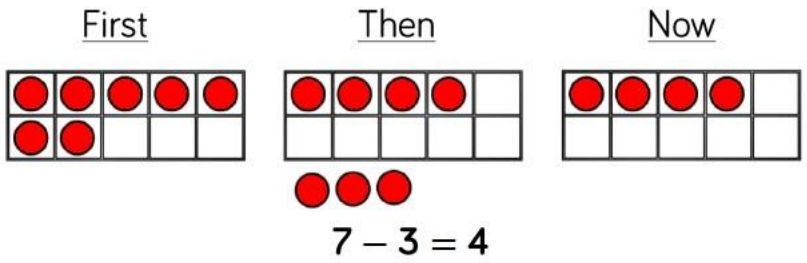
Ten Frames (within 10)



$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

4 is a part.
3 is a part.
7 is the whole.





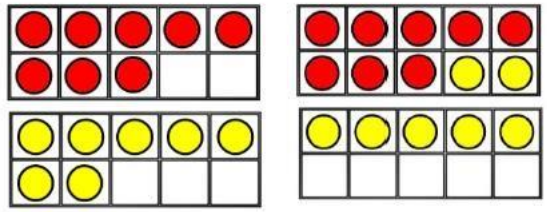
Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

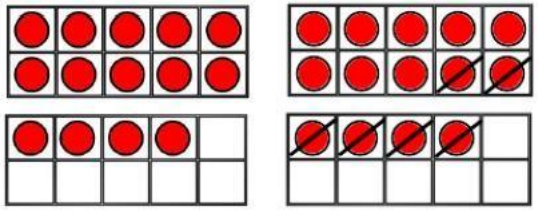
Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Ten Frames (within 20)



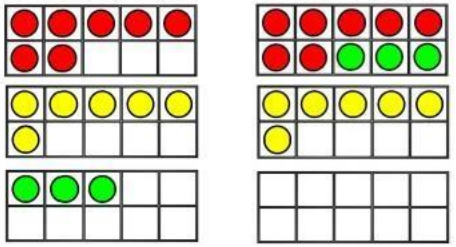
$8 + 7 = 15$

2 5



$14 - 6 = 8$

4 2



$7 + 6 + 3 = 16$

10

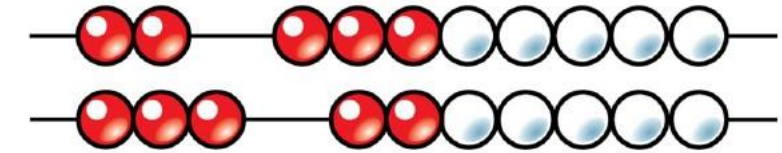
Benefits

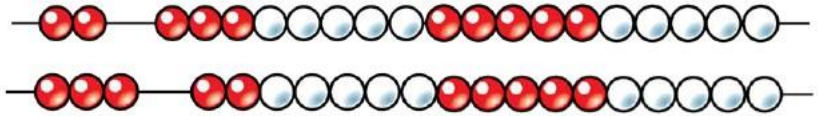
When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

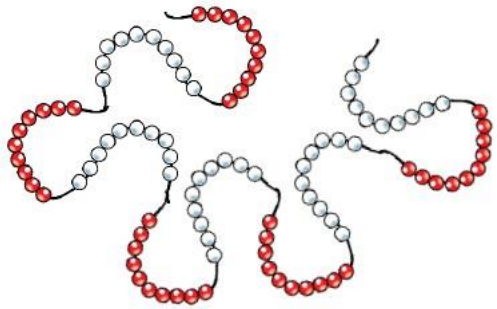
When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Bead Strings







Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

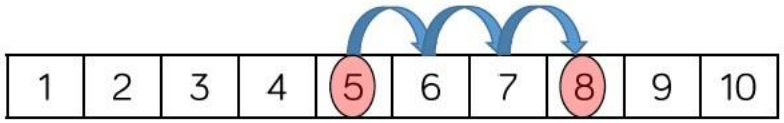
Bead strings to 10 are very effective at helping children to investigate number bonds up to 10.
They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. $2 + 8 = 10$, move one bead, $3 + 7 = 10$.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

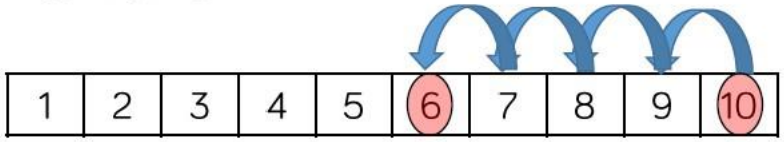
Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

Number Tracks

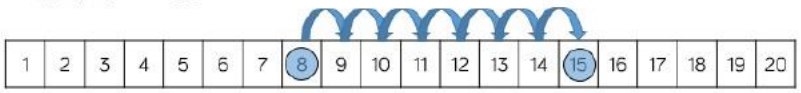
5 + 3 = 8



10 − 4 = 6



8 + 7 = 15



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

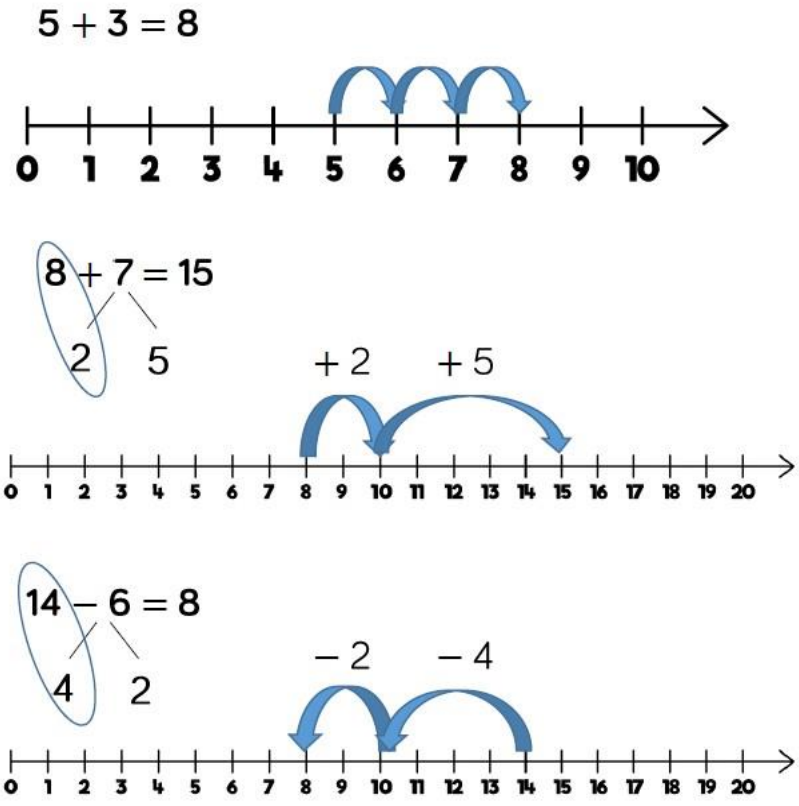
When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

Number Lines (labelled)



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

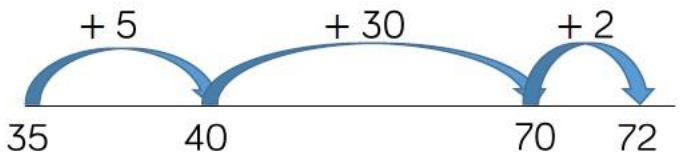
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

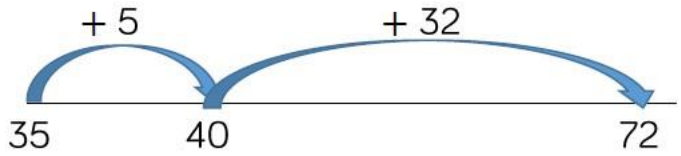
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)

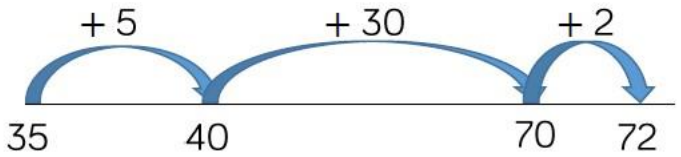
35 + 37 = 72



35 + 37 = 72



72 − 35 = 37



Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

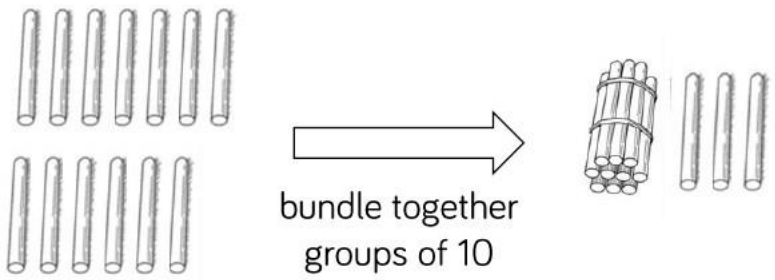
Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

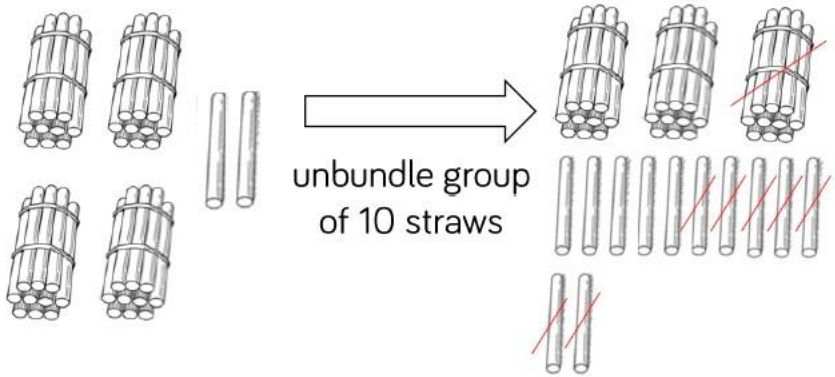
Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Straws

$7 + 6 = 13$



$42 - 17 = 25$



Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

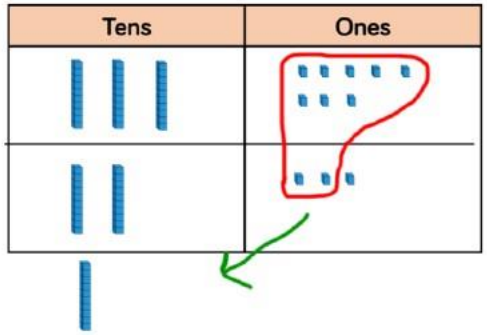
Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

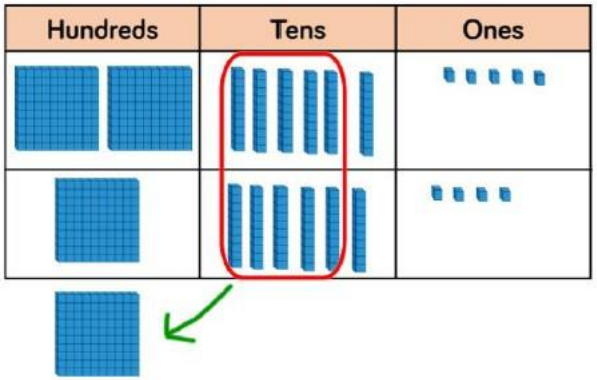
When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

Base 10/Dienes (addition)



$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline 1 \end{array}$$



$$\begin{array}{r} 265 \\ +\ 164 \\ \hline 429 \\ \hline 1 \end{array}$$

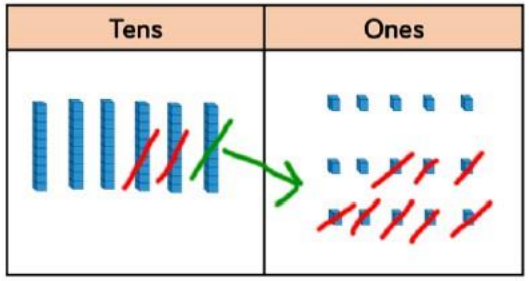
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

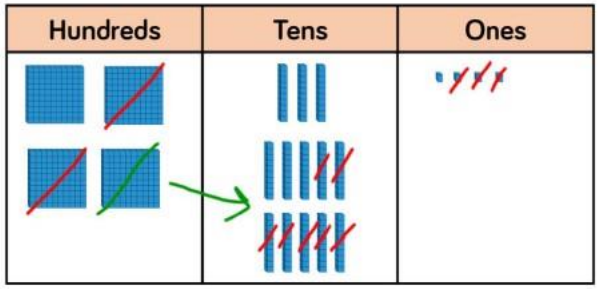
Children should first add without an exchange before moving on to addition with exchange.. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.
How many ones are there altogether?
Can we make an exchange? (Yes or No)
How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
How many ones do we have left? (Write in ones column)
Repeat for each column.

Base 10/Dienes (subtraction)



$$\begin{array}{r} {}^{5}\not{6}\,{}^{1}\not{5} \\ -\ 2\ 8 \\ \hline 3\ 7 \end{array}$$



$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}\not{3}\,{}^{1}\not{5} \\ -\ 2\ 7\ 3 \\ \hline 2\ 6\ 2 \end{array}$$

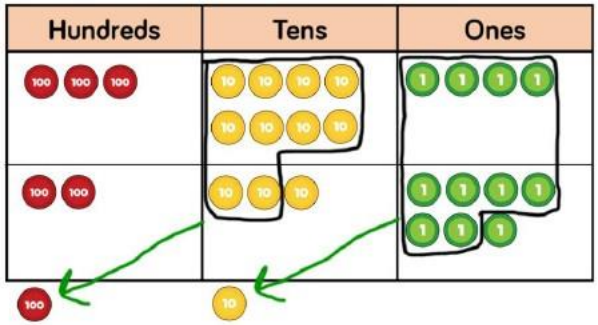
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

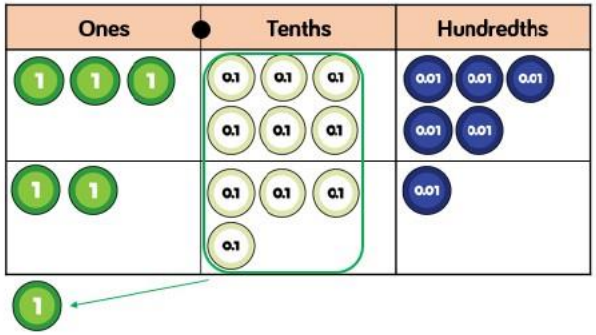
Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Place Value Counters (addition)



$$\begin{array}{r} 384 \\ +\ 237 \\ \hline 621 \\ \hline 1\ \ 1 \end{array}$$



$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

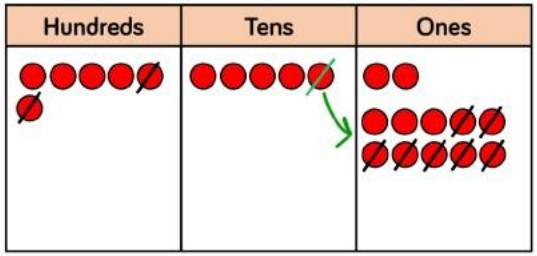
Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

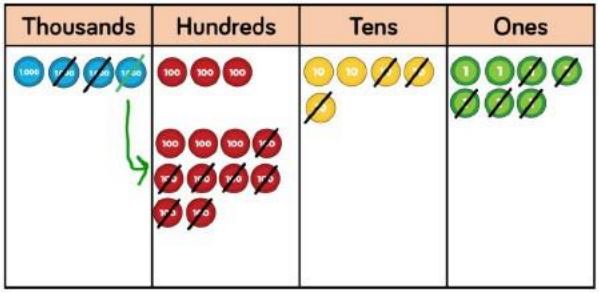
Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Place Value Counters (Subtraction)



$$\begin{array}{r} {}^{4}\not{6}\,{}^{1}\not{5}2 \\ -\ 207 \\ \hline 445 \end{array}$$



$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

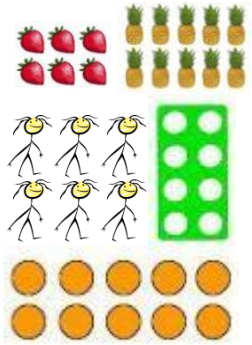
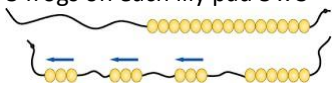
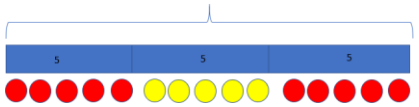
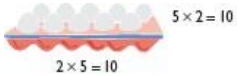
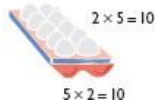
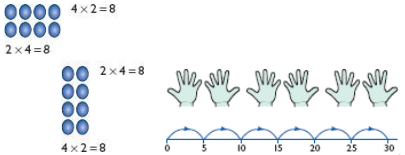
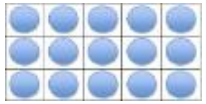
Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

Multiplication and Division

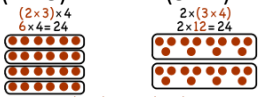
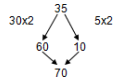
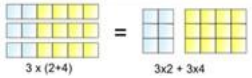
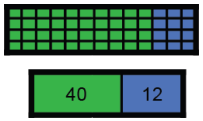
Multiplication- KS1

EYFS	**Reception: ELG 2020** • Have an understanding of number to 10, linking names of numbers, numerals, their value, and their position in the counting order. • Subitise (recognise quantities without counting) up to 5. • Automatically recall number bonds for numbers 0-5 and *for 10*, including corresponding partitioning facts. • Automatically recall double facts up 5+5 • Compare sets of objects up to 10 in different contexts, considering size and difference • Explore patterns of numbers within numbers up to 10, including evens and odds.	
Year	1	2
Layers of vocabulary  **Appendix** Beck's Tiers of Vocabulary	**Basic to subject specific (Beck's Tiers):** count in ones, twos… tens… array, groups of, equal groups odd, even **Instructional vocabulary:** carry on, continue repeat what comes next? find, choose, collect use, make, build tell me, describe, pick out, talk about, explain, show me, read, write, record	**Basic to subject specific (Beck's Tiers):** lots of, groups of ×, times, multiply, multiplied by multiple of once, twice, three times… ten times… times as (big, long, wide… and so on) repeated addition array row, column double, halve share, share equally **Instructional vocabulary:** carry on, continue, repeat, what comes next? predict describe the pattern describe the rule find, find all, find different, investigate

NC 2014	Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.		Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs.	
	Concrete, pictorial, abstract		Concrete, pictorial, abstract	
Developing Conceptual/ Procedural Understanding	**Grouping**  2 frogs on each lily pad GROUPING ITP Pictures to show 2 groups of 3 or 3 groups of 2 etc. **Doubles** 	**Arrays** (rectangular arrangements to show equal groups) 	**Repeated addition**  Introduce the x symbol once repeated addition is understood. **Grouping**  5 frogs on each lily pad 5 x 3 = 15   **Building Tables** Build tables using counting stick- forwards and backwards and with missing jumps 	**Commutativity**    5 x2 = 2 x 5  5 + 5 + 5 + 5 + 5 + 5 = 30 5 × 6 = 30 5 multiplied by 6 6 groups of 5 6 hops of 5 **Decision making** How many number sentences can you write to describe this array? Can you use addition, multiplication and division? Explain your answers. 

Multiplication- KS2

KS1	Pupils should memorise and reason with numbers in 2, 5 and 10 times tables. They should see ways to represent odd and even numbers and know how they are represented in tables. This will help them to understand the pattern in numbers. Pupils should begin to understand multiplication as scaling in terms of double and half (e.g. that tower of cubes is double the height of the other tower). Commutative law shown on array. Repeated addition can be shown mentally on a number line. Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups.	
Year	3	4
Layers of vocabulary  **Appendix** Beck's Tiers of Vocabulary	**Basic to subject specific (Beck's Tiers):** lots of, groups of ×, times, multiply, multiplication, multiplied by multiple of, product once, twice, three times… ten times… times as (big, long, wide… and so on) repeated addition array row, column double, halve share, share equally one each, two each, three each… **Instructional vocabulary:** carry on, continue repeat what comes next? predict describe the pattern, describe the rule find, find all, find different, investigate choose, decide, collect	**Basic to subject specific (Beck's Tiers):** lots of, groups of times, multiply, multiplication, multiplied by multiple of, product once, twice, three times… ten times… times as (big, long, wide… and so on) repeated addition array row, column double, halve, factor, multiple **Instructional vocabulary:** carry on, continue, repeat what comes next? predict describe the pattern, describe the rule pattern, puzzle, calculate, calculation, mental calculation, method, jotting, answer right, correct, wrong what could we try next? how did you work it out? number sentence sign, operation, symbol, equation
NC 2014	Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including 2 digit numbers times 1 digit numbers progressing to formal written methods.	Multiply 2 digit and 3 digit numbers by a 1 digit number using formal written layout. Solve problems involving multiplying and adding.

Developing Conceptual/ Procedural Understanding	**Building tables** For example, build tables using counting stick- forwards and backwards and with missing jumps **Using known facts** If 3 x 2 = 6, then 30 x 2 = 60, 60 ÷3 = 20 and 30 = 60 ÷ 2. **Associativity** (2 x 3) x 4 = 2 x (3 x 4) 	**Partitioning strategy to double** Double 35  **Place value materials to represent calculations** **Partitioning** Informal recording of partitioned numbers 15 x 5 = 75 10 x 5 = 50 5 x 5 = 25 27 x 3 = 81 20x3 = 60 7x3 = 21 "20 multiplied by 3 equals 60 and 7 multiplied by 3 equals 21. 60 add 21 equals 81."	**Grid method** 23 x 8 = 20 x 8 =160 3 x 8 = 24 23 x 8= 184 	x	20	3
---	---	---
8		| **Short multiplication** Expanded 23 x 8 24 (8 x3) 160 (8 x20) 184 leading to compact 23 x 8 184 2 **Representing problems** A group of aliens live on Planet Xert. Tinions have three legs, Quinions have four legs. The group has 22 legs	**Building tables** For example, build tables using counting stick-forwards and backwards and with missing jumps **Using known facts** If 2 x 3 = 6 then 200 x 3 = 600 and 600 ÷3 = 200 **Distributivity** 3 x (2 +4) = 3 x 2 + 3 x 4 So the '3' can be 'distributed' across the '2 + 4' into 3 times 2 and 3 times 4  leading to 13 x 4 = 10 x 4 + 3 x 4 = 52 	**Place value materials to represent calculations** **Grid method** (if needed for conceptual understanding) 346 x 9 | x	300	40	6
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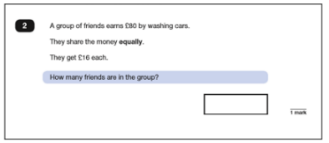
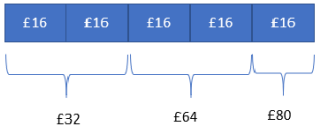
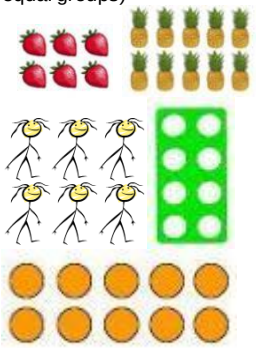
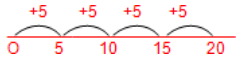
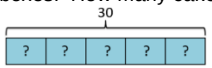
9				 **Short multiplication** Expanded 346 x 9 54 (9 x 6) 360 (9 x 40) 2700 (9 x 300) 3114 leading to compact 346 x 9 3114 4 5	**Representing problems** Multiply a number by itself and then make one factor one more and the other one less. What do you notice? Does this always happen? Eg 4 x 4 = 16 6 x 6= 36 5 x 3 = 15 7 x 5= 35 Try out more examples to prove your thinking.   Place <, >, or = in these number sentences to make them correct: 50 x 4 ▭ 4 x 50 4 x 50 ▭ 40 x 5 200 x 5 ▭ 3 x 300

			altogether. How many Tinions and Quinions might there be? Is there more than one solution?			

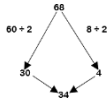
Division- KS1

EYFS	**Reception: ELG 2020** • Have an understanding of number to 10, linking names of numbers, numerals, their value, and their position in the counting order. • Subitise (recognise quantities without counting) up to 5. • Automatically recall number bonds for numbers 0-5 and *for 10*, including corresponding partitioning facts. • Automatically recall double facts up 5+5 • Compare sets of objects up to 10 in different contexts, considering size and difference • Explore patterns of numbers within numbers up to 10, including evens and odds.	
Year	1	2
Layers of vocabulary  **Appendix** Beck's Tiers of Vocabulary	**Basic to subject specific (Beck's Tiers):** count in ones, twos… tens… share, groups of, equal groups odd, even **Instructional vocabulary:** count out, share out, left, left over	**Basic to subject specific (Beck's Tiers):** share, share equally one each, two each, three each… group in pairs, threes… tens equal groups of ÷, divide, divided by, divided into left, left over **Instructional vocabulary:** tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of… show how you
NC 2014	solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs.
	Concrete, pictorial, abstract	Concrete, pictorial, abstract

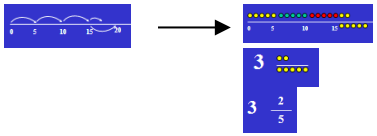
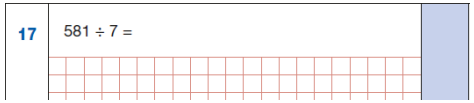
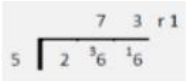
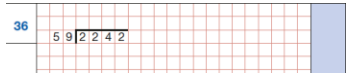
Developing Conceptual/ Procedural Understanding	**Grouping/Sharing models** Using practical contexts and cross-curricular links (PE) such as socks and shoes; animals in the ark to get into groups. Sharing models such as sharing pieces of fruit. Sharing into equal groups 6 frogs shared equally between 2 lily pads gives 3 frogs on each lily pad or Grouping in equal groups 6 frogs grouped in 2s need 3 lily pads to sit on   GROUPING ITP How many twos?	**Arrays** (rectangular arrangements to show equal groups)  **Decision making** How many cars can you make if you have 8 wheels?  How many different ways can you arrange 12 buttons in equal groups? 	**Grouping/Sharing models** Introduce the ÷ symbol  15 frogs shared equally between three lily pads 15 ÷3 = 5 or 15 frogs grouped in 5s need 3 lily pads to sit on 15 ÷5 = 3 15 ÷ 3 = 5 groups of 3 (grouping)  20 ÷ 2 = 10   5 hops in 15. How big is each hop? There are 7 cakes and 2 children. How many cakes will they get each? (Leftovers/remainders introduced) 7 ÷ 2 = 3r1	**Arrays representing the dividend**  10 ÷ 2 = 5 and 10 ÷ 5 = 2 **Repeated addition (to reach a given target)**  There are 20 sweets in a bag. How many children can have 5 each?  **Repeated subtraction (from a given quantity)**  **Links to tables** Use language of division linked to tables using counting stick **Representing problems** Jane has 30 cakes. She wants to share them equally between 5 boxes. How many cakes should go in each box?  30 ÷ 5 = 6 Number of cakes in each box = 6

Division- KS2

KS1	Noticing how counting in multiples if 2, 5 and 10 relates to the number of groups you have counted (introducing times tables) links to division. An understanding of the more you share between, the less each person will get (e.g. would you prefer to share these grapes between 2 people or 3 people? Why?) Secure understanding of grouping means you count the number of groups you have made. Whereas sharing means you count the number of objects in each group.	
Year	3	4
Layers of vocabulary  **Appendix** Beck's Tiers of Vocabulary	**Basic to subject specific (Beck's Tiers):** share, share equally one each, two each, three each… group in pairs, threes… tens equal groups of ÷, divide, division, divided by, divided into left, left over, remainder, dividend, divisor **Instructional vocabulary:** calculate, work out, solve, investigate question, answer, check	**Basic to subject specific (Beck's Tiers):** share, share equally one each, two each, three each… group in pairs, threes… tens equal groups of ÷, divide, division, divided by, divided into left, left over, remainder, dividend, divisor **Instructional vocabulary:** calculate, work out, solve, investigate, question, answer, check
NC 2014	Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including 2 digit numbers times 1 digit numbers progressing to formal written methods.	Practise to become fluent in the formal written method of short division with exact answers.

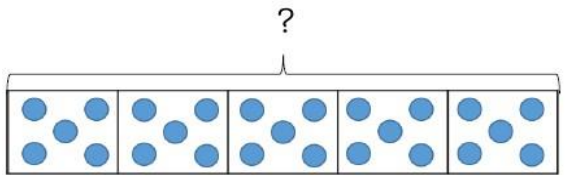
Developing Conceptual/ Procedural Understanding	**Links to tables**  For example, use language of division linked to tables using counting stick **Using known facts** If 3 x 2 = 6, then 30 x 2 = 60, 60 ÷3 = 20 and 30 = 60 ÷ 2. **Partitioning strategy to halve** Halve 68  **Rearranging the dividend to find multiples of the divisor.** 48 ÷ 3 = 'What do I know about the 3 x tables?' "I know 3 x 10 = 30 and 3 x 6 = 18."  48 ÷ 3 = 16	**Place value materials to represent calculations** **Short division** 72 ÷ 3 = 2 4 3 7 ^{1}2 '72 divided by 3. 7 tens shared equally between 3 is 2 with a remainder of 1 ten. Exchange the 1 ten for 10 units. I now have 12 units which shared equally between 3 is 4. The answer is 24." **Representing problems** Andy says 'I can use my three times table to work out 180 ÷ 3'. Explain what Andy could do to work out this calculation.	**Links to tables**  For example, use language of division linked to tables using counting stick **Using known facts** If 2 x 3 = 6 then 200 x 3 = 600 and 600 ÷3 = 200 **Rearranging the dividend to find multiples of the divisor.** 69÷ 3 = 'What do I know about the 3 x tables?' "I know 3 x 10 = 30 and 3 x 3 = 9."  69 ÷ 3 = 23 2 4 r 1 3 7 ^{1}3	**Place value materials to represent calculations** **Short division** 372 ÷ 6 = 6 2 6 3 7 ^{1}2 '372 divided by 6. 3 hundreds cannot be shared equally between 6, so exchange the hundreds for 30 tens. I now have 37 tens which shared equally between 6 is 6 with a remainder of 1 ten. Exchange the ten for 10 units. I now have 12 units which shared equally between 6 is 2. The answer is 62." **Representing problems** Alan says that the solution to 186 ÷ 4 can be written as '46 remainder 2' or as '46.5'. Do you agree? Explain your answer.

Year	5	6
Layers of vocabulary  **Appendix** Beck's Tiers of Vocabulary	**Basic to subject specific (Beck's Tiers):** equal groups of divide, division, divided by, divided into remainder factor, quotient, divisible by inverse **Instructional vocabulary:** calculate, work out, solve, investigate question, answer, check same, different missing number/s number facts, number pairs, number bonds greatest value, least value	**Basic to subject specific (Beck's Tiers):** equal groups of divide, division, divided by, divided into remainder factor, quotient, divisible by inverse, remainders as fractions or decimals **Instructional vocabulary:** calculate, work out, solve, investigate question, answer, check same, different missing number/s number facts, number pairs, number bonds greatest value, least value
NC 2014	Divide numbers up to 4 digits by a 1 digit number using the formal written method of short division and interpret remainders appropriately for the context (as remainders, as fractions, as decimals or by rounding, e.g. 98 ÷ 4 = = 24 r2 = 24 ½= 24.5 ≈ 25). Solve problems involving multiplication and division including using knowledge of factors and multiples, squares and cubes. Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign. Solve problems involving multiplication and division including scaling by simple fractions and problems involving simple rates.	Divide numbers up to 4 digits by a 2 digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate to the context. Divide numbers up to 4 digits by a 2 digit number using the formal written method of short division where appropriate, interpreting remainders according to the context. Solve problems involving addition, subtraction, multiplication and division.

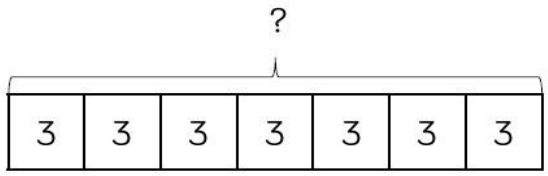
Developing Conceptual/ Procedural Understanding	**Using known facts** If 6 ÷ 2 = 3 then 6000 ÷ 2 = 3000 and 6000 ÷ 20 = 300 **Place value materials to represent calculations** **Short division** 483 ÷ 7 = 6 9 r1 7 4 ^{4}8 ^{6}4 "484 divided by 7. 4 hundreds cannot be shared equally between 7, so exchange the hundreds for 40 tens. I now have 48 tens which shared equally between 7 is 6 with a remainder of 6 tens. Exchange the 6 tens for 60 units, we now have 64 units. 64 shared equally between 7 equals 9 remainder 1. The answer is 69 r1."	**<u>Interpreting remainders</u>** 17 ÷ 5 "What do I know? 17 is not a multiple of 5."  3 $\frac{2}{5}$ = 3.4 From knowledge of decimal/fraction equivalents or by converting into . Examples:  581 ÷ 7 could be calculated by the formal written method of short division or it could be calculated by rearranging the dividend, using known facts, into 560 and 21. **Representing problems** Correct the errors in the calculation below. Explain the error. 266 ÷ 5 = 73.1  	**Using known facts** If 6 ÷ 2 = 3 then 6÷ 0.2 = 30 and 6 ÷ 0.02 = 300 **Short division** 97.6 ÷ 5 = 1 9 . 5 2 5 9 ^{4}7. ^{2}6^{1}0 "97.6 divided by 5. 9 tens shared equally between 5 is 1 with a remainder of 4 tens. Exchange the ten for 10 units. I now have 47 units which shared equally between 5 is 9 with a remainder of 2 units. Exchange the 2 units for 20 tenths, we now have 26 tenths. 26 shared equally between 5 equals 5 with a remainder of 1 tenth. Extend the dividend with a 0 in the hundredths column. Exchange the tenth for 10 hundredths. 10 shared equally between 5 equals 2. The answer is 19.52." **Long division** (thinking not generally recorded) 384 ÷ 16	 With questions of this type where the divisor is close to a number linked to the times tables, encourage the children to use known facts and adjustment to set up the partial tables. **Representing problems** Megan divides 500 by 8 and gets the answer 62r4. She re writes it as 62 r 1/2 . Is she right? Explain your answer. Using factors to simplify long division 25) 815

			"What do I know about the divisor?" Record partial tables. **24** **16 (38 tens ÷16 = 2** **384 r6; 2 x 16 =32)** **-32 (bring the 4 down)** **(64 units ÷ 16 =4)** **64** **- 64** **(no remainder)** **0**	$5\overline{)815}\ 165$ $5\overline{)165}\ 35$ **Simplify the fractions for remainders**

Bar Model

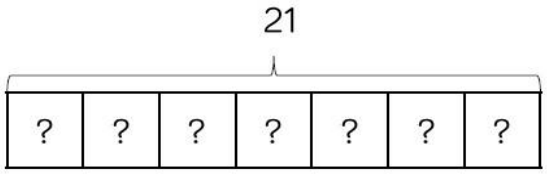


$5 \times 5 = 25$

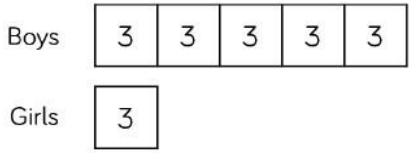


$3 \times 7 = 21$

$7 \times 3 = 21$



$21 \div 7 = 3$



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

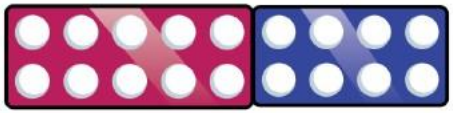
It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
The multiple bar model provides an opportunity to compare the groups.

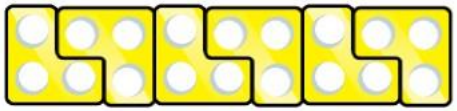
Number Shapes

$5 \times 4 = 20$
$4 \times 5 = 20$

$5 \times 4 = 20$
$4 \times 5 = 20$



$18 \div 3 = 6$



Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd × odd = even, odd × even = odd, even × even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.

Bead Strings

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 3 = 5$

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 5 = 3$

$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

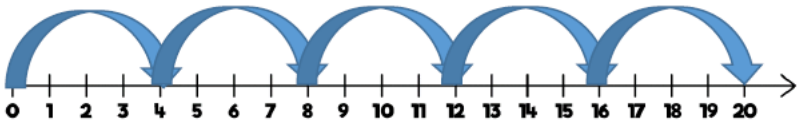
Benefits

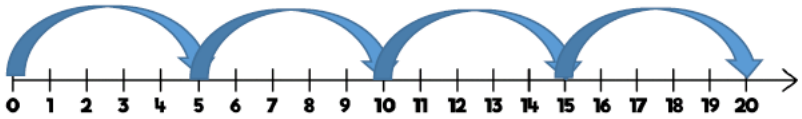
Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.
Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

Number Lines (labelled)





$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

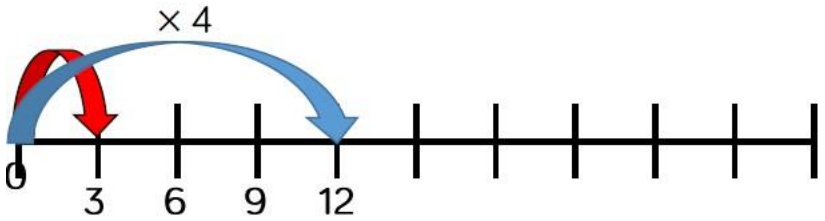
Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

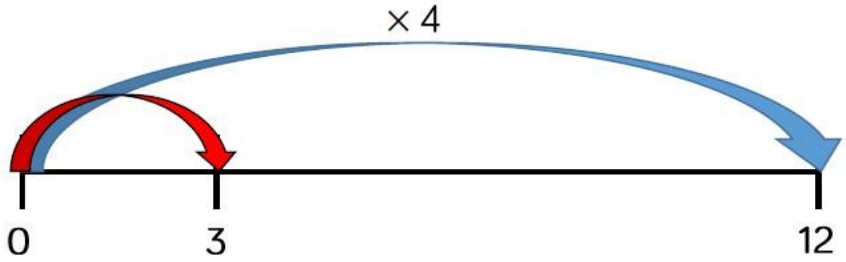
When multiplying, children start at 0 and then count on to find the product of the numbers.
When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

Number Lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

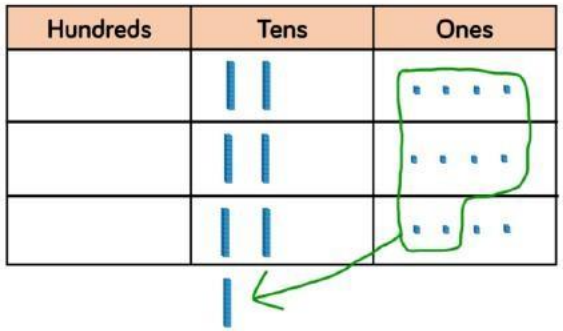
Benefits

Children can use blank number lines to represent scaling as multiplication or division.

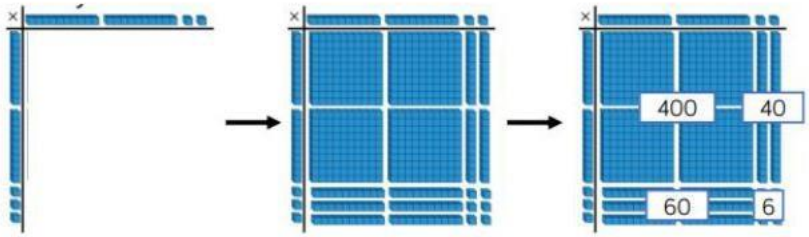
Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

Base 10/Dienes (multiplication)



$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \\ \hline 1 \end{array}$$



Benefits

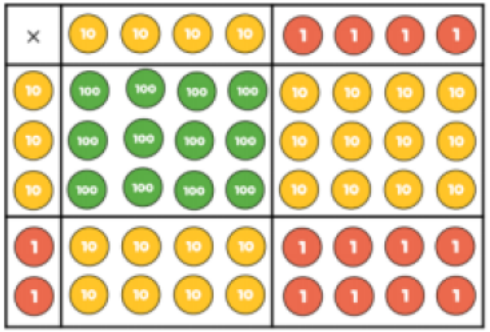
Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

Place Value Counters (multiplication)

$$\begin{array}{r} 34 \\ \times\ 5 \\ \hline 170 \\ \hline 1\ 2 \end{array}$$



$$\begin{array}{r} 44 \\ \times\ 32 \\ \hline 8 \\ 80 \\ 120 \\ +\ 1200 \\ \hline 1408 \\ \hline \end{array}$$

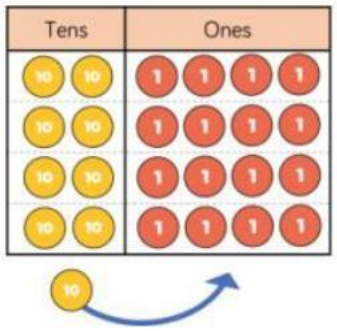
Benefits

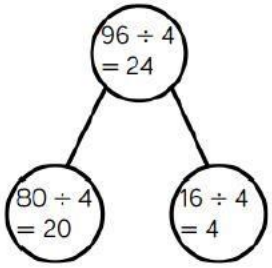
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

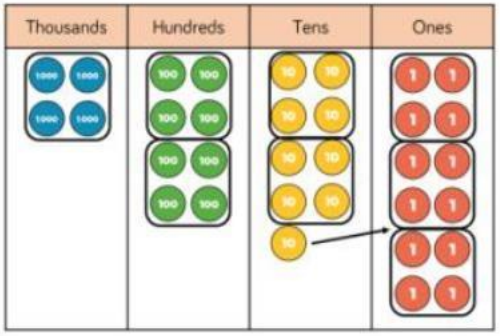
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

Place Value Counters (division)







$$\begin{array}{r} 1223 \\ 4\overline{)\,4892^{1}} \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.